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Polarized J/ψ from χ_{cJ} and ψ' Decays at the Tevatron

Bernd A. Kniehl and Jungil Lee
II. Institut für Theoretische Physik, Universität Hamburg,
Luruper Chaussee 149, 22761 Hamburg, Germany

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Abstract

We calculate the partonic cross sections for the hadroproduction of polarized χ_{cJ} mesons using the factorization formalism of nonrelativistic QCD. We also describe how the polarization is passed on to the J/ψ mesons in the radiative feed-down $\chi_{cJ} \to J/\psi + \gamma$ and in the double-cascade decays of ψ' mesons via χ_{cJ} intermediate states. These represent the missing ingredients needed to predict the polarization of prompt J/ψ mesons at the Fermilab Tevatron.

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1 Introduction

Since its discovery in 1974, the J/ψ meson has provided a useful laboratory for quantitative tests of quantum chromodynamics (QCD) and, in particular, of the interplay of perturbative and nonperturbative phenomena. The factorization formalism of nonrelativistic QCD (NRQCD) [1] provides a rigorous theoretical framework for the description of heavy-quarkonium production and decay. This formalism implies a separation of short-distance coefficients, which can be calculated perturbatively as expansions in the strong-coupling constant α_s, from long-distance matrix elements (ME's), which must be extracted from experiment. The relative importance of the latter can be estimated by means of velocity scaling rules, *i.e.* the ME's are predicted to scale with a definite power of the heavy-quark (Q) velocity v in the limit $v \to 0$. In this way, the theoretical predictions are organized as double expansions in α_s and v. A crucial feature of this formalism is that it takes into account the complete structure of the $Q\overline{Q}$ Fock space, which is spanned by the states ${}^{2S+1}L_J^{(a)}$ with definite spin S, orbital angular momentum L, total angular momentum J, and colour multiplicity $a = \underline{1}, \underline{8}$. In particular, this formalism predicts the existence of colour-octet processes in nature. This means that $Q\overline{Q}$ pairs are produced at short distances in colour-octet states and subsequently evolve into physical (colour-singlet) quarkonia by the nonperturbative emission of soft gluons. The greatest triumph of this formalism was that it was able to correctly describe the cross section of inclusive charmonium hadroproduction measured in $p\overline{p}$ collisions at the Fermilab Tevatron [2], which had turned out to be more than one order of magnitude in excess of the theoretical prediction based on the traditional colour-singlet model (CSM) [3,4].

The *experimentum crucis* of the NRQCD factorization formalism is the measurement of the polarization of charmonium with quantum numbers $J^{PC} = 1^{--}$, *i.e.* J/ψ and ψ' mesons, in direct hadroproduction, which is predicted to be fully transverse at sufficiently large values of the transverse momentum (p_T) [5]. A convenient measure of the polarization is provided by the variable

$$\alpha = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}, \tag{1}$$

where σ_T and σ_L are the transverse and longitudinal components of the total cross section $\sigma = \sigma_T + \sigma_L$, respectively. The unpolarized case is characterized by $\alpha = 0$, while $\alpha = 1$ and -1 correspond to fully transverse and longitudinal polarizations, respectively. As for the J/ψ and ψ' mesons, α is best measured by analyzing the angular distributions of their decays to $\mu^+\mu^-$ pairs, which are proportional to $1 + \alpha\cos^2\theta^*$, where θ^* is the angle between the μ^+ direction in the charmonium rest frame and the charmonium direction in the laboratory frame. The first such measurement was recently performed for ψ' mesons by the CDF Collaboration at the Tevatron [6]. Unfortunately, the statistics are still too low to allow for a meaningful test of the NRQCD prediction [7,8]. In the same experiment, the polarization was also measured for prompt J/ψ mesons, *i.e.* those which do not stem from the decays of bottom-flavored hadrons, with about a hundred times more statistics. This makes it necessary to include in the theoretical prediction the production of polarized J/ψ

mesons via the decays of the heavier charmonium states χ_{c1}, χ_{c2}, and ψ', which are directly produced. In fact, these feed-down channels make up approximately 15%, 15%, and 10% of the prompt-J/ψ signal, respectively. There is an additional, yet smaller contribution to prompt J/ψ production from the double-cascade decays of directly produced ψ' mesons via χ_{cJ} intermediate states. The purpose of this paper is to provide analytic results for the missing ingredients which are necessary to make quantitative predictions for all these contributions. The resulting prediction for α has already been presented in Ref. [9]. It is consistent with the CDF data at intermediate values of p_T, but it disagrees by about three standard deviations from the data point of largest p_T.

In order to predict α for prompt J/ψ mesons to lowest order (LO) in the NRQCD factorization formalism, we need the polarized cross sections for the partonic subprocesses $a + b \to c\overline{c}(n) + d$, where $a, b, d = g, q, \overline{q}$ with $q = u, d, s$, for $n = {}^3S_1^{(1,8)}$, ${}^3P_J^{(1,8)}$, and ${}^1S_0^{(8)}$. The corresponding results for the unpolarized case are well established [3,4,10]. The polarized cross sections may be found in Refs. [8,11], with the exception of those for the P-wave colour-singlet $c\overline{c}$ Fock states $n = {}^3P_1^{(1)}$ and ${}^3P_2^{(1)}$, which are relevant for the χ_{c1} and χ_{c2} mesons. These missing results could have already been obtained in the CSM, before the advent of the NRQCD factorization formalism. However, the calculation is somewhat tedious, especially for the spin-two case of χ_{c2}. The calculation is further complicated by the fact that α is measured in the hadronic center-of-mass (CM) frame, so that one cannot benefit from the usual simplifications which occur when one chooses the partonic CM frame. Instead, the cross section does not only depend on the partonic Mandelstam variables, but also on the boost factor relating the partonic and hadronic CM frames. Therefore, the helicity amplitude method, which provides a powerful tool for the treatment of complicated partonic subprocesses, such as $g + g \to \chi_{c2} + g$ without polarization, cannot be directly employed. In Refs. [7,8,11], a Lorentz-covariant formalism for describing the polarization of massive spin-one particles in the hadronic CM frame was elaborated. We adopt this formalism and extend it to the spin-two case. Furthermore, we need the probabilities for the transitions from the $c\overline{c}$ Fock states of the χ_{c1}, χ_{c2}, and ψ' mesons to those of the J/ψ meson in the presence of polarization. Owing to heavy-quark spin symmetry, they are trivial for the single-cascade decay of the ψ' meson. The results for the ${}^3P_1^{(1)}$ and ${}^3P_2^{(1)}$ channels may be found in Ref. [12], where gluon fragmentation to polarized χ_{c1} and χ_{c2} mesons with subsequent radiative feed-down to polarized J/ψ mesons was studied. The missing pieces of information are provided here.

This paper is organized as follows. In Sec. 2, we present a general Lorentz-covariant formalism which allows us to conveniently describe the partonic cross sections for the inclusive production of massive spin-one and spin-two bosons, the helicities of which are fixed in the hadronic CM frame. In Sec. 3, we decompose the cross section for the prompt production of longitudinal J/ψ mesons into the direct, single-cascade, and double cascade contributions, and we explain how the various components factorize into cross sections of partonic subprocesses $a + b \to c\overline{c}(n) + d$ with definite $c\overline{c}$ helicity, scalar ME's, and measured branching fractions. In Sec. 4, we present separate theoretical predictions of α for J/ψ mesons from the single-cascade decays of χ_{c1}, χ_{c2}, and ψ' mesons, and from the double-cascade decays of ψ' mesons via χ_{cJ} intermediate states. Our conclusions are

summarized in Sec. 5. In Appendix A, we present the ingredients which enter the analytic expressions for the polarized cross sections of the partonic subprocesses $a+b \to c\overline{c}(n)+d$, with $n = {}^3P_1^{(1)}$ and ${}^3P_2^{(1)}$, which represent the missing links for the theoretical prediction of α from prompt J/ψ mesons.

2 Covariant Tensor Decomposition of Polarized Cross Sections

The partonic cross sections for the production of massive spin-one and spin-two bosons, the polarizations of which are to be measured in the hadronic CM frame, depend not only on the partonic Mandelstam variables, but also on the boost factor relating the partonic and hadronic CM frames. A Lorentz-covariant formalism to deal with this problem in the spin-one case was elaborated in Refs. [7,8,11]. It can be applied to the direct production of polarized J/ψ, χ_{c1}, and ψ' mesons. In order to also include χ_{c2} mesons in the polarization analysis, we need to generalize this formalism to the spin-two case. Prior to doing that, we recall the spin-one formalism.

In the following, we work in the hadronic CM frame. Let $\epsilon^\mu(\lambda)$ be the polarization four-vector of a spin-one boson H with mass M, four-momentum P, and helicity λ. In the case of longitudinal polarization ($\lambda = 0$), a Lorentz-covariant expression reads [7,9,11]:

$$\epsilon^\mu(0) = Z^\mu = \frac{(P\cdot Q/M)P^\mu - MQ^\mu}{\sqrt{(P\cdot Q)^2 - M^2 s}}, \tag{2}$$

where Q is the total four-momentum of the colliding hadrons and $s = Q^2$. It is convenient to decompose the polarization sum,

$$\rho^{\mu\nu} = \sum_{\lambda=-1}^{1} \epsilon^{\mu *}(\lambda)\epsilon^\nu(\lambda) = -g^{\mu\nu} + \frac{P^\mu P^\nu}{M^2}, \tag{3}$$

into longitudinal and transverse components as

$$\rho^{\mu\nu} = \sum_{|\lambda|=0}^{1} \rho^{\mu\nu}_{|\lambda|}, \tag{4}$$

where

$$\rho^{\mu\nu}_{|\lambda|} = \sum_{\lambda=\pm|\lambda|} \epsilon^{\mu *}(\lambda)\epsilon^\nu(\lambda). \tag{5}$$

Specifically, we have

$$\rho^{\mu\nu}_0 = Z^\mu Z^\nu, \quad \rho^{\mu\nu}_1 = \rho^{\mu\nu} - \rho^{\mu\nu}_0. \tag{6}$$

Let $\mathcal{M}$ be the transition matrix element of the partonic subprocess

$$a(p_a) + b(p_b) \to H(P,\lambda) + d, \tag{7}$$

where the four-momenta and helicities given in parentheses refer to the hadronic CM frame. If we average the absolute square of $\mathcal{M}$ over the spin and colour states of the initial-state partons a and b, and sum over $\lambda = \pm|\lambda|$ and the spin and colour states of the final-state parton d, we can write the outcome as

$$\overline{|\mathcal{M}|^2_{|\lambda|}} = S_{\mu\nu}\rho^{\mu\nu}_{|\lambda|}, \tag{8}$$

where $S^{\mu\nu}$ is a rank-two Lorentz tensor which depends on p_a, p_b, and P. Exploiting the the symmetry and orthogonality properties $\rho^{\mu\nu}_{|\lambda|} = \rho^{\nu\mu}_{|\lambda|}$ and $P_\mu\rho^{\mu\nu}_{|\lambda|} = 0$, respectively, we can decompose $S^{\mu\nu}$ as

$$S^{\mu\nu} = F\sum_{i=1}^{4} c_i s_i^{\mu\nu}, \tag{9}$$

where the common factor F and the coefficients c_i are scalar functions of the partonic Mandelstam variables

$$\begin{aligned}\hat{s} &= (p_a + p_b)^2,\\ \hat{t} &= (p_a - P)^2,\\ \hat{u} &= (p_b - P)^2,\end{aligned} \tag{10}$$

which satisfy the relation $\hat{s} + \hat{t} + \hat{u} = M^2$, and

$$\begin{aligned}s_1^{\mu\nu} &= g^{\mu\nu},\\ s_2^{\mu\nu} &= p_a^\mu p_a^\nu,\\ s_3^{\mu\nu} &= p_b^\mu p_b^\nu,\\ s_4^{\mu\nu} &= p_a^\mu p_b^\nu.\end{aligned} \tag{11}$$

We now turn to the spin-two case. Let $\epsilon^{\mu\nu}(\lambda)$ be the polarization tensor of a spin-two boson H with mass M, four-momentum P, and helicity λ. It can be constructed from the polarization four-vectors of two spin-one bosons with the same mass and four-momentum with the help of the addition theorem for two angular momenta as [13]

$$\epsilon^{\mu\nu}(\lambda) = \sum_{\lambda_1,\lambda_2=-1}^{1} \langle 1,\lambda_1;1,\lambda_2|2,\lambda\rangle \epsilon^\mu(\lambda_1)\epsilon^\nu(\lambda_2), \tag{12}$$

where $\langle 1,\lambda_1;1,\lambda_2|2,\lambda\rangle$ are Clebsch-Gordon coefficients. Similarly to Eq. (4), we decompose the polarization sum,

$$\rho^{\mu\nu\rho\sigma} = \sum_{\lambda=-2}^{2} \epsilon^{\mu\nu *}(\lambda)\epsilon^{\rho\sigma}(\lambda) = \frac{1}{2}\left(\rho^{\mu\rho}\rho^{\nu\sigma} + \rho^{\mu\sigma}\rho^{\nu\rho}\right) - \frac{1}{3}\rho^{\mu\nu}\rho^{\rho\sigma}, \tag{13}$$

as

$$\rho^{\mu\nu\rho\sigma} = \sum_{|\lambda|=0}^{2} \rho^{\mu\nu\rho\sigma}_{|\lambda|}, \tag{14}$$

where

$$\rho_{|\lambda|}^{\mu\nu\rho\sigma} = \sum_{\lambda=\pm|\lambda|} \epsilon^{\mu\nu*}(\lambda)\epsilon^{\rho\sigma}(\lambda). \tag{15}$$

Specifically, we have [12]

$$\begin{aligned}
\rho_0^{\mu\nu\rho\sigma} &= \frac{1}{6}\left(2\rho_0^{\mu\nu} - \rho_1^{\mu\nu}\right)\left(2\rho_0^{\rho\sigma} - \rho_1^{\rho\sigma}\right), \\
\rho_1^{\mu\nu\rho\sigma} &= \frac{1}{2}\left(\rho_0^{\mu\rho}\rho_1^{\nu\sigma} + \rho_0^{\mu\sigma}\rho_1^{\nu\rho} + \rho_0^{\nu\rho}\rho_1^{\mu\sigma} + \rho_0^{\nu\sigma}\rho_1^{\mu\rho}\right), \\
\rho_2^{\mu\nu\rho\sigma} &= \frac{1}{2}\left(\rho_1^{\mu\rho}\rho_1^{\nu\sigma} + \rho_1^{\mu\sigma}\rho_1^{\nu\rho} - \rho_1^{\mu\nu}\rho_1^{\rho\sigma}\right).
\end{aligned} \tag{16}$$

We now consider the case when H in Eq. (7) represents a spin-two boson. Then, Eq. (8) is replaced by

$$\overline{|\mathcal{M}|^2_{|\lambda|}} = S_{\mu\nu\rho\sigma}\rho_{|\lambda|}^{\mu\nu\rho\sigma}, \tag{17}$$

where $S^{\mu\nu\rho\sigma}$ is a rank-four Lorentz tensor which depends on p_a, p_b, and P. Making use of the symmetry, orthogonality, and tracelessness properties $\rho_{|\lambda|}^{\mu\nu\rho\sigma} = \rho_{|\lambda|}^{\rho\sigma\mu\nu} = \rho_{|\lambda|}^{\nu\mu\rho\sigma}$, $P_\mu\rho_{|\lambda|}^{\mu\nu\rho\sigma} = 0$, and $g_{\mu\nu}\rho_{|\lambda|}^{\mu\nu\rho\sigma} = 0$, respectively, we can constrain the decomposition of $S^{\mu\nu\rho\sigma}$ to be of the form

$$S^{\mu\nu\rho\sigma} = F\sum_{i=1}^{10} c_i s_i^{\mu\nu\rho\sigma}, \tag{18}$$

where F and c_i are scalar functions of $\hat{s}$, $\hat{t}$, and $\hat{u}$, defined in Eq. (10), and

$$\begin{aligned}
s_1^{\mu\nu\rho\sigma} &= g^{\mu\rho}g^{\nu\sigma}, \\
s_2^{\mu\nu\rho\sigma} &= g^{\mu\rho}p_a^\nu p_a^\sigma, \\
s_3^{\mu\nu\rho\sigma} &= g^{\mu\rho}p_b^\nu p_b^\sigma, \\
s_4^{\mu\nu\rho\sigma} &= g^{\mu\rho}p_a^\nu p_b^\sigma, \\
s_5^{\mu\nu\rho\sigma} &= p_a^\mu p_a^\nu p_a^\rho p_a^\sigma, \\
s_6^{\mu\nu\rho\sigma} &= p_b^\mu p_b^\nu p_b^\rho p_b^\sigma, \\
s_7^{\mu\nu\rho\sigma} &= p_a^\mu p_a^\nu p_a^\rho p_b^\sigma, \\
s_8^{\mu\nu\rho\sigma} &= p_b^\mu p_b^\nu p_b^\rho p_a^\sigma, \\
s_9^{\mu\nu\rho\sigma} &= p_a^\mu p_a^\nu p_b^\rho p_b^\sigma, \\
s_{10}^{\mu\nu\rho\sigma} &= p_a^\mu p_b^\nu p_a^\rho p_b^\sigma.
\end{aligned} \tag{19}$$

The dependence on the reference frame of the measurement enters through Eq. (2) and resides in the factors $s_{i\mu\nu}\rho_{|\lambda|}^{\mu\nu}$ and $s_{i\mu\nu\rho\sigma}\rho_{|\lambda|}^{\mu\nu\rho\sigma}$, which are independent of the partonic subprocess. These factors explicitly depend on the longitudinal-momentum fractions which the partons a and b receive from the hadrons from which they spring. Their calculation is straightforward, and we refrain from listing them here. The functions F and c_i in Eqs. (9) and (18) are independent of the hadron momenta and the reference frame; they only depend on the partonic subprocess. The set of these functions which are relevant for the direct hadroproduction of polarized $^3S_1$ charmonium ($H = J/\psi, \psi'$) in the NRQCD

factorization formalism may be found in Refs. [8,11]. This includes the ${}^3S_1^{(8)}$ channel, which also contributes in the cases $H=\chi_{c1},\chi_{c2}$. In Appendix A, we provide the missing sets of functions pertinent to the latter two cases, namely those associated with the ${}^3P_1^{(1)}$ and ${}^3P_2^{(1)}$ channels, respectively.

3 Polarization in Cascade

We now explain how to calculate the polarization variable α, defined in Eq. (1), for prompt J/ψ production in the NRQCD factorization formalism. Our approach not only applies to hadroproduction, but also to any other production mechanism, such as photoproduction, deep-inelastic scattering, e^+e^- annihilation, two-photon scattering, *etc.* It is convenient to calculate α from the unpolarized cross section σ and its longitudinal component σ_L. Prompt J/ψ mesons originate from direct J/ψ production, from single-cascade decays of directly produced χ_{cJ} and ψ' mesons, and from double-cascade decays of directly produced ψ' mesons via χ_{cJ} intermediate states. The unpolarized cross section $\sigma^{\text{prompt } J/\psi}$ is simply obtained by adding the unpolarized cross sections of the various direct-production processes multiplied with the appropriate branching fractions.

The calculation of the longitudinal cross section $\sigma_L^{\text{prompt } J/\psi}$ is more involved. In the following, we explain how it can be expressed in terms of the reduced partonic cross sections $\hat{\sigma}_{|\lambda|}(n)$ for the production of $c\bar{c}$ pairs in Fock states n with definite absolute helicities $|\lambda|$, scalar ME's, and branching fractions. We have

$$\sigma_L^{\text{prompt } J/\psi}=\sigma_L^{\text{direct } J/\psi}+\sigma_L^{\chi_{cJ}}+\sigma_L^{\psi'}+\sigma_L^{\psi'\to\chi_{cJ}}. \tag{20}$$

The cross sections for the direct production of polarized J/ψ and ψ' mesons is given by

$$\sigma_{L,T}^{\text{direct } \psi}=\sum_n \hat{\sigma}_{0,1}(n)\left\langle\mathcal{O}^{\psi}(n)\right\rangle, \tag{21}$$

where the summation is over $n={}^3S_1^{(1)}$, ${}^3S_1^{(8)}$, ${}^1S_0^{(8)}$, and ${}^3P_J^{(8)}$. The relevant reduced partonic cross sections $\hat{\sigma}_{0,1}(n)$ may be extracted from Refs. [8,11]. Equation (21) allows us to obtain the first term in Eq. (20).

In the NRQCD factorization formalism, the radiative decay $\chi_{cJ}\to J/\psi+\gamma$ is approximately treated as an electric dipole transition. Due to heavy-quark spin symmetry, the helicity of the J/ψ meson is then given by the third spin component m_S of the χ_{cJ} meson. Thus, angular-momentum conservation constrains the transition probability of a χ_{cJ} meson with helicity m_J to a longitudinally polarized J/ψ meson, with $m_S=0$, to be proportional to $\sum_{m_L=-1}^{1}|\langle 1,m_L;1,0|J,m_J\rangle|^2$. By applying this kind of analysis to the leading $c\bar{c}$ Fock states $n={}^3P_J^{(1)}$ and ${}^3S_1^{(8)}$ relevant for the χ_{cJ} mesons, we find the second term in Eq. (20) to be

$$\sigma_L^{\chi_{cJ}}=\left[\frac{1}{3}\hat{\sigma}_0\left({}^3P_0^{(1)}\right)\left\langle\mathcal{O}^{\chi_{c0}}\left({}^3P_0^{(1)}\right)\right\rangle+\frac{1}{3}\hat{\sigma}_0\left({}^3S_1^{(8)}\right)\left\langle\mathcal{O}^{\chi_{c0}}\left({}^3S_1^{(8)}\right)\right\rangle\right]B(\chi_{c0}\to J/\psi+\gamma)$$

$$+\left\{\frac{1}{2}\hat{\sigma}_1\left({}^3P_1^{(1)}\right)\left\langle\mathcal{O}^{\chi_{c0}}\left({}^3P_0^{(1)}\right)\right\rangle+\left[\frac{1}{2}\hat{\sigma}_0\left({}^3S_1^{(8)}\right)+\frac{1}{4}\hat{\sigma}_1\left({}^3S_1^{(8)}\right)\right]\left\langle\mathcal{O}^{\chi_{c0}}\left({}^3S_1^{(8)}\right)\right\rangle\right\}$$
$$\times B(\chi_{c1}\to J/\psi+\gamma)+\left\{\left[\frac{2}{3}\hat{\sigma}_0\left({}^3P_2^{(1)}\right)+\frac{1}{2}\hat{\sigma}_1\left({}^3P_2^{(1)}\right)\right]\left\langle\mathcal{O}^{\chi_{c0}}\left({}^3P_0^{(1)}\right)\right\rangle\right.$$
$$\left.+\left[\frac{17}{30}\hat{\sigma}_0\left({}^3S_1^{(8)}\right)+\frac{13}{60}\hat{\sigma}_1\left({}^3S_1^{(8)}\right)\right]\left\langle\mathcal{O}^{\chi_{c0}}\left({}^3S_1^{(8)}\right)\right\rangle\right\}B(\chi_{c2}\to J/\psi+\gamma). \tag{22}$$

Here, we made use of the multiplicity relations

$$\left\langle\mathcal{O}^{\chi_{cJ}}\left({}^3P_J^{(1)}\right)\right\rangle=(2J+1)\left\langle\mathcal{O}^{\chi_{c0}}\left({}^3P_0^{(1)}\right)\right\rangle,$$
$$\left\langle\mathcal{O}^{\chi_{cJ}}\left({}^3S_1^{(8)}\right)\right\rangle=(2J+1)\left\langle\mathcal{O}^{\chi_{c0}}\left({}^3S_1^{(8)}\right)\right\rangle, \tag{23}$$

which follow to leading order in v^2 from heavy-quark spin symmetry. To leading order in v^2, the absolute value of the first derivative of the radial wave function of P-wave charmonium at the origin $|R_P'(0)|$, which is one of the input parameters of the CSM, is related to $\left\langle\mathcal{O}^{\chi_{c0}}\left({}^3P_0^{(1)}\right)\right\rangle$ by $\left\langle\mathcal{O}^{\chi_{c0}}\left({}^3P_0^{(1)}\right)\right\rangle = 3N_c\,|R_P'(0)|\,/(2\pi)$, where $N_c = 3$. The combinatorial factors multiplying the terms in Eq. (22) which involve $\hat{\sigma}_{|\lambda|}\left({}^3P_J^{(1)}\right)$ agree with those found in Ref. [12].

The hadronic decay $\psi'\to J/\psi$ proceeds predominantly through a double chromoelectric dipole transition. Since transitions involving a spin flip are suppressed and the recoil momentum of the J/ψ meson is negligible, the J/ψ meson has the same helicity as the ψ' meson. Thus, the third term in Eq. (20) may be evaluated as

$$\sigma_L^{\psi'}=\sigma_L^{\text{direct }\psi'}B(\psi'\to J/\psi+X). \tag{24}$$

The fourth term in Eq. (20) may be derived in a similar fashion as the contribution to $\sigma_L^{\chi_{cJ}}$ from the ${}^3S_1^{(8)}$ channel, and it reads

$$\sigma_L^{\psi'\to\chi_{cJ}}=\frac{1}{3}\sigma_L^{\text{direct }\psi'}B(\psi'\to\chi_{c0}+\gamma)B(\chi_{c0}\to J/\psi+\gamma)$$
$$+\left(\frac{1}{2}\sigma_L^{\text{direct }\psi'}+\frac{1}{4}\sigma_T^{\text{direct }\psi'}\right)B(\psi'\to\chi_{c1}+\gamma)B(\chi_{c1}\to J/\psi+\gamma)$$
$$+\left(\frac{17}{30}\sigma_L^{\text{direct }\psi'}+\frac{13}{60}\sigma_T^{\text{direct }\psi'}\right)B(\psi'\to\chi_{c2}+\gamma)B(\chi_{c2}\to J/\psi+\gamma). \tag{25}$$

4 Numerical Analysis

We are now in a position to explore the phenomenological implications of the analytic results presented in Secs. 2 and 3, and Appendix A for the inclusive hadroproduction of polarized J/ψ mesons at the Tevatron, with CM energy $\sqrt{s} = 1.8$ TeV. As in the CDF analysis [6], we consider the p_T distribution $d\sigma/dp_T$, integrated over the rapidity interval $|y| \leq 0.6$. We work at leading order in the NRQCD factorization formalism. We use the MRST98LO parton density functions (PDF's) of the proton [14] and evaluate $\alpha_s(\mu)$ from the one-loop formula with $\Lambda_{\text{QCD}} = 174$ MeV [14]. We set the renormalization

and factorization scales μ equal to the transverse mass $m_T = \sqrt{4m_c^2 + p_T^2}$, with $m_c = 1.5$ GeV. We adopt the nonperturbative ME's appropriate for our choice of PDF's from Table I of Ref. [9]. As for the J/ψ and ψ' mesons, fits to the Tevatron data do not simultaneously constrain $\left\langle \mathcal{O}^\psi \left({}^1S_0^{(8)}\right)\right\rangle$ and $\left\langle \mathcal{O}^\psi \left({}^3P_0^{(8)}\right)\right\rangle$, but only the linear combination $M_r = \left\langle \mathcal{O}^\psi \left({}^1S_0^{(8)}\right)\right\rangle + r\left\langle \mathcal{O}^\psi \left({}^3P_0^{(8)}\right)\right\rangle/m_c^2$, with some optimal value of r. For simplicity, we assume that $\left\langle \mathcal{O}^\psi \left({}^1S_0^{(8)}\right)\right\rangle = r\left\langle \mathcal{O}^\psi \left({}^3P_0^{(8)}\right)\right\rangle/m_c^2 = M_r/2$. We adopt the values of the relevant branching fractions from Ref. [15]. We essentially work within the fusion picture, where the $c\bar{c}$ bound state is formed in the primary hard-scattering process. In the high-p_T regime, only those partonic subprocesses survive where the $c\bar{c}$ pair is created from a single gluon which is close to its mass shell. These contributions, which contain large logarithms of the form $\ln(m_T/2m_c)$, are conveniently implemented by using fragmentation functions (FF's). These logarithms are then resummed by evolving the FF's from their starting scale $\mu = 2m_c$ up to the scale $\mu = m_T$. In the case of the unpolarized cross section $\sigma^{\text{prompt } J/\psi}$, we approximately include this resummation by multiplying the fusion cross section $\hat{\sigma}\left({}^3S_1^{(8)}\right)$ with the ratio of the corresponding fragmentation cross sections with final-state factorization scales $\mu = m_T$ and $\mu = 2m_c$. In the case of the longitudinal cross section $\sigma_L^{\text{prompt } J/\psi}$, we include the LO fragmentation cross sections $\hat{\sigma}_0(n)$ with $n = {}^3S_1^{(8)}$, ${}^1S_0^{(8)}$, and ${}^3P_J^{(8)}$ [16], which are of order α_s^4. These contributions are particularly important because the corresponding fusion cross sections vanish to order α_s^3. Further details concerning the implementation of the fragmentation contributions may be found in Ref. [9].

In Fig. 1, we study the p_T dependence of the polarization variable α for prompt J/ψ mesons. For comparison, we also show α for the various production mechanisms which contribute to the prompt-J/ψ signal, namely direct J/ψ production, single-cascade decays of directly produced χ_{cJ} and ψ' mesons, and double-cascade decays of directly produced ψ' mesons via χ_{cJ} intermediate states. A quantitative understanding of the asymptotic large-p_T behaviour of α may be obtained by observing that, in this limit, inclusive charmonium production dominantly proceeds via the gluon fragmentation process $g \to c\bar{c}\left({}^3S_1^{(8)}\right)$ [17]. Since large-p_T gluons are almost on shell and thus predominantly transversely polarized, $\hat{\sigma}_1\left({}^3S_1^{(8)}\right)$ should greatly exceed all other reduced partonic cross sections at large p_T values [18]. It hence follows that, for directly produced J/ψ mesons, α approaches unity in the large-p_T limit [5]. By heavy-quark spin symmetry, the same is true for J/ψ mesons from single-cascade decays of ψ' mesons [9]. The situation is more involved for J/ψ mesons from the radiative feed-down of χ_{cJ} mesons and for those from double-cascade decays of ψ' mesons. Inspecting Eqs. (22) and (25) in the large-p_T limit, we can derive the asymptotic expression

$$\alpha = \frac{60x_0 + 15x_1 + 21x_2}{60x_0 + 75x_1 + 73x_2}, \tag{26}$$

where $x_J = (2J+1)B(\chi_{cJ} \to J/\psi+\gamma)$ in the first case and $x_J = B(\psi' \to \chi_{cJ}+\gamma)B(\chi_{cJ} \to J/\psi + \gamma)$ in the second one. Experimental data suggests that $B(\chi_{c0} \to J/\psi + \gamma) \ll B(\chi_{c1} \to J/\psi + \gamma) \approx 2B(\chi_{c2} \to J/\psi + \gamma)$ and that $B(\psi' \to \chi_{cJ} + \gamma)$ is approximately independent of J [15]. We thus obtain $\alpha^{\chi_{cJ}} \approx 39/163 \approx 0.24$ and $\alpha^{\psi' \to \chi_{cJ}} \approx 51/223 \approx$

0.23. (Without these approximations, we find $\alpha^{\chi_{cJ}} \approx 0.24$ and $\alpha^{\psi' \to \chi_{cJ}} \approx 0.24$.) From Fig. 1 we see that the corresponding predictions at $p_T = 30$ GeV are already very close to these asymptotic values. Finally, we observe that the predicted p_T distribution of α from prompt J/ψ mesons is relatively close to the prediction for direct J/ψ mesons. As is evident from Fig. 1, this is a consequence of a strong cancellation between the $\chi_{cJ} \to J/\psi + \gamma$ and $\psi' \to J/\psi + X$ channels, which could not have been anticipated without explicit calculation.

5 Conclusions

The very recent measurement of the polarization of prompt J/ψ mesons at the Tevatron [6] offers a unique opportunity to test the NRQCD factorization formalism [1], which postulates the existence of color-octet processes in nature. The theoretical prediction of prompt-J/ψ polarization is complicated by the contributions due to the feed-down from heavier charmonium states, which come in addition to the direct-J/ψ contribution and make up about 40% of the prompt-J/ψ rate. Specifically, one needs to consider the single-cascade decays of directly produced χ_{c1}, χ_{c2}, and ψ' mesons, and the double-cascade decays of directly produced ψ' mesons via χ_{cJ} intermediate states. In this paper, we provided, in analytic form, the missing ingredients which are necessary to make this prediction. These include the cross sections of the partonic subprocesses $a + b \to c\overline{c}(n) + d$ with $n = {}^3P_1^{(1)}$ and ${}^3P_2^{(1)}$, and the probabilities for the transitions from the $c\overline{c}$ Fock states of the χ_{c1}, χ_{c2}, and ψ' mesons to those of the J/ψ meson in the presence of polarization. We presented quantitative predictions, in terms of the polarization variable α, for the various production mechanisms which contribute to the prompt-J/ψ signal. In the large-p_T limit, α approaches the asymptotic values $\alpha^{\text{direct } J/\psi} = \alpha^{\psi'} = 1$ and $\alpha^{\chi_{cJ}} \approx \alpha^{\psi' \to \chi_{cJ}} \approx 0.24$.

A comprehensive discussion of α for prompt J/ψ mesons together with a conservative analysis of the theoretical errors may be found in Ref. [9]. The theoretical prediction [9] is consistent with the CDF data [6] at intermediate values of p_T, but it exceeds the data point of largest p_T by about three standard deviations of the latter. For the time being, it is premature to conclude that this represents experimental evidence against the NRQCD factorization formalism. On the theoretical side, the prediction should be improved by the inclusion of next-to-leading-order corrections in α_s and v^2, which are not yet available. This could have an appreciable effect, since $\alpha_s(2m_c) \approx v^2 \approx 0.3$ is not actually so small against unity. On the experimental side, Run II of the Tevatron is expected to increase the presently available charmonium data sample by roughly a factor of 20. This will allow for the polarization to be measured with higher precision and out to larger values of p_T. Furthermore, it should then be feasible to discriminate between the various channels which constitute the prompt-J/ψ signal, as is presently done in the measurement of the unpolarized cross section. Should the discrepancy persist despite concerted theoretical and experimental progress, then one possible explanation could be that the NRQCD factorization formalism fails for charmonium simply because m_c is too small. Other effects, such as the one due to nonvanishing intrinsic transverse momenta of

the initial-state gluons [19], might be important. It is, therefore, worthwhile to perform parallel studies for polarized bottomonium.

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A Partonic Cross Sections for Color-Singlet Production of Polarized χ_{c1} and χ_{c2} Mesons

In this appendix, we present the ingredients which are necessary to describe the direct hadroproduction of polarized χ_{c1} and χ_{c2} mesons via the color-singlet channels $n = {}^3P_1^{(1)}$ and ${}^3P_2^{(1)}$, respectively. The reduced differential cross sections $d\hat{\sigma}_{|\lambda|}/d\hat{t}$ of the partonic subprocesses $a + b \to c\overline{c}(n) + d$, where the $c\overline{c}$ pair has absolute helicity $|\lambda|$, are given by

$$\frac{d\hat{\sigma}_{|\lambda|}}{d\hat{t}} = \frac{1}{16\pi\hat{s}^2}\overline{|\mathcal{M}|^2_{|\lambda|}}. \tag{A.1}$$

Here, the partons a, b, and d are treated as massless, while the charm quark c has mass m_c. In the spin-one (spin-two) case, $\overline{|\mathcal{M}|^2_{|\lambda|}}$ is given by Eq. (8) (Eq. (17)), which contains the process-dependent tensor $S^{\mu\nu}$ ($S^{\mu\nu\rho\sigma}$). The functions F and c_i which appear in the Lorentz decomposition of the latter according to Eq. (9) (Eq. (18)) are listed below. As in Eq. (22), the color-singlet ME $\left\langle \mathcal{O}^{\chi_{c0}}\left({}^3P_0^{(1)}\right)\right\rangle$ has been factored out. Summing over $|\lambda|$, we recover the well-known results for the unpolarized case [4]. As a further check for our analytical analysis, we also verified the differential cross sections of the partonic subprocesses $a+b \to c\overline{c}\left({}^3P_0^{(1)}\right) + d$, relevant for the hadroproduction of χ_{c0} mesons, given in Ref. [4].

$q\overline{q} \to c\overline{c}\left({}^3P_1^{(1)}\right) + g$:

$$\begin{aligned}
F &= -\frac{64(4\pi\alpha_s)^3}{81 m_c(\hat{t}+\hat{u})^4}, \\
c_1 &= \hat{t}\hat{u}, \\
c_2 &= 2\hat{t}, \\
c_3 &= 2\hat{u}, \\
c_4 &= 2(\hat{t}+\hat{u}).
\end{aligned} \tag{A.2}$$

$q\overline{q} \to c\overline{c}\left({}^3P_2^{(1)}\right) + g$:

$$\begin{aligned}
F &= \frac{128(4\pi\alpha_s)^3 m_c}{81\hat{s}^2(\hat{t}+\hat{u})^4}, \\
c_1 &= \hat{s}(\hat{t}+\hat{u})^2, \\
c_2 &= 4\left[\hat{s}^2+\hat{t}^2+2\hat{s}(\hat{t}+\hat{u})\right], \\
c_3 &= 4\left[\hat{s}^2+\hat{u}^2+2\hat{s}(\hat{t}+\hat{u})\right], \\
c_4 &= 8\left[\hat{s}^2-\hat{t}\hat{u}+2\hat{s}(\hat{t}+\hat{u})\right], \\
c_5 &= c_6 = 8\hat{s}, \\
c_7 &= c_8 = 16\hat{s}, \\
c_9 &= -16(\hat{t}+\hat{u}), \\
c_{10} &= 16(\hat{s}+\hat{t}+\hat{u}).
\end{aligned} \tag{A.3}$$

$gq \to c\overline{c}\left({}^3P_1^{(1)}\right) + q$:

$$\begin{aligned}
F &= \frac{8(4\pi\alpha_s)^3}{27m_c(\hat{s}+\hat{u})^4}, \\
c_1 &= \hat{s}\hat{u}, \\
c_2 &= 2\hat{s}, \\
c_3 &= 0, \\
c_4 &= 2(\hat{s}-\hat{u}).
\end{aligned} \tag{A.4}$$

$gq \to c\overline{c}\left({}^3P_2^{(1)}\right) + q$:

$$\begin{aligned}
F &= -\frac{16(4\pi\alpha_s)^3 m_c}{27\hat{t}^2(\hat{s}+\hat{u})^4}, \\
c_1 &= \hat{t}(\hat{s}+\hat{u})^2, \\
c_2 &= 4\left[\hat{s}^2+\hat{t}^2+2\hat{t}(\hat{s}+\hat{u})\right], \\
c_3 &= 4\hat{s}^2\hat{u}^2, \\
c_4 &= 8\hat{s}(\hat{s}+\hat{u}), \\
c_5 &= 8\hat{t}, \\
c_6 &= c_8 = 0, \\
c_7 &= 16\hat{t}, \\
c_9 &= -16(\hat{s}+\hat{u}), \\
c_{10} &= 16(\hat{s}+\hat{t}+\hat{u}).
\end{aligned} \tag{A.5}$$

$gg \to c\overline{c}\left({}^3P_1^{(1)}\right) + g$:

$$F = \frac{(4\pi\alpha_s)^3}{6m_c(\hat{s}+\hat{t})^4(\hat{t}+\hat{u})^4(\hat{u}+\hat{s})^4},$$

$$
\begin{aligned}
c_1 &= -2\hat{s}^2\hat{t}^2\hat{u}^2(\hat{s}\hat{t}+\hat{t}\hat{u}+\hat{u}\hat{s})(\hat{s}^2+\hat{t}^2+\hat{u}^2-2\hat{s}\hat{t}-2\hat{t}\hat{u}-2\hat{u}\hat{s}) \\
&\quad -\hat{s}^2\hat{t}^2\hat{u}^2(\hat{s}+\hat{t}+\hat{u})\left[\hat{s}\hat{t}(\hat{s}+\hat{t})+\hat{t}\hat{u}(\hat{t}+\hat{u})+\hat{u}\hat{s}(\hat{u}+\hat{s})\right] \\
&\quad -(\hat{s}\hat{t}+\hat{t}\hat{u}+\hat{u}\hat{s})\left[\hat{s}^3\hat{t}^3(\hat{s}-\hat{t})^2+\hat{t}^3\hat{u}^3(\hat{t}-\hat{u})^2+\hat{u}^3\hat{s}^3(\hat{u}-\hat{s})^2\right], \\
c_2 &= 2(\hat{s}+\hat{t})^2\Big\{\hat{s}^2(\hat{t}+\hat{u})\left[\hat{s}(\hat{s}-\hat{t})(\hat{s}-\hat{u})(\hat{t}+\hat{u})-\hat{t}\hat{u}(\hat{t}-\hat{u})^2\right] \\
&\quad +(\hat{t}-\hat{u})\left[\hat{t}^2+\hat{u}^2-\hat{s}(\hat{t}+\hat{u})\right]\left[\hat{s}^2\hat{t}^2+\hat{t}^2\hat{u}^2+\hat{u}^2\hat{s}^2+\hat{s}\hat{t}\hat{u}(\hat{s}+2\hat{t})\right]\Big\}, \\
c_3 &= c_2|_{\hat{t}\leftrightarrow\hat{u}}, \\
c_4 &= 4(\hat{t}+\hat{u})^2\Big\{\hat{s}^3(\hat{s}^4+\hat{t}^4+\hat{u}^4)-\hat{s}\hat{t}\hat{u}\left[\hat{s}^4+\hat{s}^2(5\hat{t}^2-7\hat{t}\hat{u}+5\hat{u}^2)+\hat{t}\hat{u}(\hat{t}^2-3\hat{t}\hat{u}+\hat{u}^2)\right]\Big\} \\
&\quad -4(\hat{t}+\hat{u})(\hat{t}-\hat{u})^2\left[\hat{s}^4(\hat{t}+2\hat{u})(2\hat{t}+\hat{u})-\hat{s}^2\hat{t}\hat{u}(\hat{t}^2+\hat{u}^2)+\hat{t}^3\hat{u}^3\right].
\end{aligned}
\tag{A.6}
$$

$gg \to c\overline{c}\left({}^3P_2^{(1)}\right)+g$:

$$
\begin{aligned}
F &= \frac{8(4\pi\alpha_s)^3 m_c}{3(\hat{s}\hat{t}\hat{u})^2(\hat{s}+\hat{t})^4(\hat{t}+\hat{u})^4(\hat{u}+\hat{s})^4}, \\
c_1 &= \hat{s}\hat{t}\hat{u}(\hat{s}+\hat{t})^2(\hat{t}+\hat{u})^2(\hat{u}+\hat{s})^2(\hat{s}^2+\hat{t}^2+\hat{u}^2)(\hat{s}\hat{t}+\hat{t}\hat{u}+\hat{u}\hat{s})^2, \\
c_2 &= 2(\hat{s}+\hat{t})^2\Big\{4\hat{u}^8\hat{s}\hat{t}(\hat{s}+\hat{t})^2+\hat{u}^7\hat{s}\hat{t}(\hat{s}+\hat{t})(7\hat{s}+26\hat{s}\hat{t}+7\hat{t}^2) \\
&\quad -\hat{u}^6\left[\hat{s}^6+\hat{t}^6-5\hat{s}\hat{t}(\hat{s}^4+\hat{t}^4)-45\hat{s}^2\hat{t}^2(\hat{s}^2+\hat{t}^2)-87\hat{s}^3\hat{t}^3\right] \\
&\quad +\hat{u}^5\hat{s}^2\hat{t}^2(\hat{s}+\hat{t})(32\hat{s}^2+71\hat{s}\hat{t}+32\hat{t}^2)+\hat{u}^4\hat{s}^2\hat{t}^2\left[10(\hat{s}^4+\hat{t}^4)+64\hat{s}\hat{t}(\hat{s}^2+\hat{t}^2)+105\hat{s}^2\hat{t}^2\right] \\
&\quad +\hat{u}^3\hat{s}^3\hat{t}^3(\hat{s}+\hat{t})(17\hat{s}^2+32\hat{s}\hat{t}+17\hat{t}^2)+\hat{u}^2\hat{s}^4\hat{t}^4(8\hat{s}^2+11\hat{s}\hat{t}+8\hat{t}^2)-\hat{u}\hat{s}^5\hat{t}^5(\hat{s}+\hat{t}) \\
&\quad -\hat{s}^6\hat{t}^6\Big\}, \\
c_3 &= c_2|_{\hat{t}\leftrightarrow\hat{u}}, \\
c_4 &= -2(\hat{s}+\hat{t})(\hat{s}+\hat{u})\Big\{\hat{s}^6(\hat{t}^2-\hat{u}^2)^2(\hat{t}+2\hat{u})(2\hat{t}+\hat{u})-\hat{s}^5\hat{t}^2\hat{u}^2(\hat{t}+\hat{u})(3\hat{t}+5\hat{u})(5\hat{t}+3\hat{u}) \\
&\quad -\hat{s}^4\hat{t}\hat{u}\left[3(\hat{t}^6+\hat{u}^6)+28\hat{t}\hat{u}(\hat{t}^4+\hat{u}^4)+83\hat{t}^2\hat{u}^2(\hat{t}^2+\hat{u}^2)+114\hat{t}^3\hat{u}^3\right] \\
&\quad -\hat{s}^3\hat{t}^2\hat{u}^2\left[11(\hat{t}^5+\hat{u}^5)+53\hat{t}\hat{u}(\hat{t}^3+\hat{u}^3)+92\hat{t}^2\hat{u}^2(\hat{t}+\hat{u})\right] \\
&\quad -\hat{s}^2\hat{t}^3\hat{u}^3(\hat{t}+\hat{u})^2(13\hat{t}^2+10\hat{t}\hat{u}+13\hat{u}^2)-\hat{s}\hat{t}^4\hat{u}^4(\hat{t}+\hat{u})(5\hat{t}^2+6\hat{t}\hat{u}+5\hat{u}^2)-2\hat{t}^6\hat{u}^6\Big\}, \\
c_5 &= 4\hat{s}\hat{t}\hat{u}^2(\hat{s}+\hat{t})^2(2\hat{s}\hat{t}+\hat{t}\hat{u}+\hat{u}\hat{s})^2\left[2\hat{u}(\hat{s}+\hat{t}+\hat{u})^2+(\hat{s}+\hat{t})(\hat{s}\hat{t}-\hat{u}^2)\right], \\
c_6 &= c_5|_{\hat{t}\leftrightarrow\hat{u}}, \\
c_7 &= 4\hat{s}\hat{t}\hat{u}^2(\hat{s}+\hat{t})\Big[\hat{t}^5(\hat{s}+\hat{u})(5\hat{s}^2+12\hat{s}\hat{u}+5\hat{u}^2)+\hat{s}^3\hat{u}^3(3\hat{s}^2+4\hat{s}\hat{u}+3\hat{u}^2) \\
&\quad +\hat{u}^5\hat{t}(11\hat{s}^2+13\hat{s}\hat{t}+5\hat{t}^2)+\hat{u}^4\hat{t}(37\hat{s}^3+68\hat{s}^2\hat{t}+43\hat{s}\hat{t}^2+8\hat{t}^3) \\
&\quad +\hat{u}^3\hat{s}\hat{t}(43\hat{s}^3+132\hat{s}^2\hat{t}+124\hat{s}\hat{t}^2+45\hat{t}^3)+\hat{u}^2\hat{s}^2\hat{t}(19\hat{s}^3+110\hat{s}^2\hat{t}+156\hat{s}\hat{t}^2+82\hat{t}^3) \\
&\quad +\hat{u}\hat{s}^3\hat{t}^2(31\hat{s}^2+83\hat{s}\hat{t}+61\hat{t}^2)+\hat{s}^4\hat{t}^3(11\hat{s}+16\hat{t})\Big], \\
c_8 &= c_7|_{\hat{t}\leftrightarrow\hat{u}}, \\
c_9 &= 4(\hat{s}+\hat{t})(\hat{s}+\hat{u})\Big\{\hat{s}^5\left[2(\hat{t}^6+\hat{u}^6)+5\hat{t}\hat{u}(\hat{t}^4+\hat{u}^4)+4\hat{t}^2\hat{u}^2(\hat{t}^2+\hat{t}\hat{u}+\hat{u}^2)\right] \\
&\quad +\hat{s}^4\hat{t}\hat{u}(\hat{t}+\hat{u})\left[3(\hat{t}^4+\hat{u}^4)-\hat{t}\hat{u}(\hat{t}+\hat{u})^2\right]-2\hat{s}^3\hat{t}^2\hat{u}^2\left[\hat{t}^4+\hat{u}^4+6\hat{t}\hat{u}(\hat{t}^2+\hat{u}^2)+9\hat{t}^2\hat{u}^2\right]
\end{aligned}
$$

$$-\hat{s}^2\hat{t}^3\hat{u}^3(\hat{t}+\hat{u})(\hat{t}^2+\hat{u}^2)+4\hat{s}\hat{t}^4\hat{u}^4(\hat{t}+\hat{u})^2+2\hat{t}^5\hat{u}^5(\hat{t}+\hat{u})\Big\}\,,$$

$$c_{10}=-4\Big\{\hat{s}^7\left[2(\hat{t}^6+\hat{u}^6)+2\hat{t}\hat{u}(\hat{t}^4+\hat{u}^4)-21\hat{t}^2\hat{u}^2(\hat{t}^2+\hat{u}^2)-46\hat{t}^3\hat{u}^3\right]$$
$$+\hat{s}^6(\hat{t}+\hat{u})\left[2(\hat{t}^6+\hat{u}^6)+4\hat{t}\hat{u}(\hat{t}^4+\hat{u}^4)-53\hat{t}^2\hat{u}^2(\hat{t}^2+\hat{u}^2)-134\hat{t}^3\hat{u}^3\right]$$
$$+\hat{s}^5\hat{t}\hat{u}\left[2(\hat{t}^6+\hat{u}^6)-43\hat{t}\hat{u}(\hat{t}^4+\hat{u}^4)-256\hat{t}^2\hat{u}^2(\hat{t}^2+\hat{u}^2)-428\hat{t}^3\hat{u}^3\right]$$
$$-\hat{s}^4\hat{t}^2\hat{u}^2(\hat{t}+\hat{u})\left[15(\hat{t}^4+\hat{u}^4)+136\hat{t}\hat{u}(\hat{t}^2+\hat{u}^2)+264\hat{t}^2\hat{u}^2\right]$$
$$-\hat{s}^3\hat{t}^3\hat{u}^3(\hat{t}+\hat{u})^2(32\hat{t}^2+93\hat{t}\hat{u}+32\hat{u}^2)-\hat{s}^2\hat{t}^4\hat{u}^4(\hat{t}+\hat{u})(19\hat{t}^2+34\hat{t}\hat{u}+19\hat{u}^2)$$
$$+2\hat{t}^6\hat{u}^6(\hat{s}+\hat{t}+\hat{u})\Big\}\,. \tag{A.7}$$

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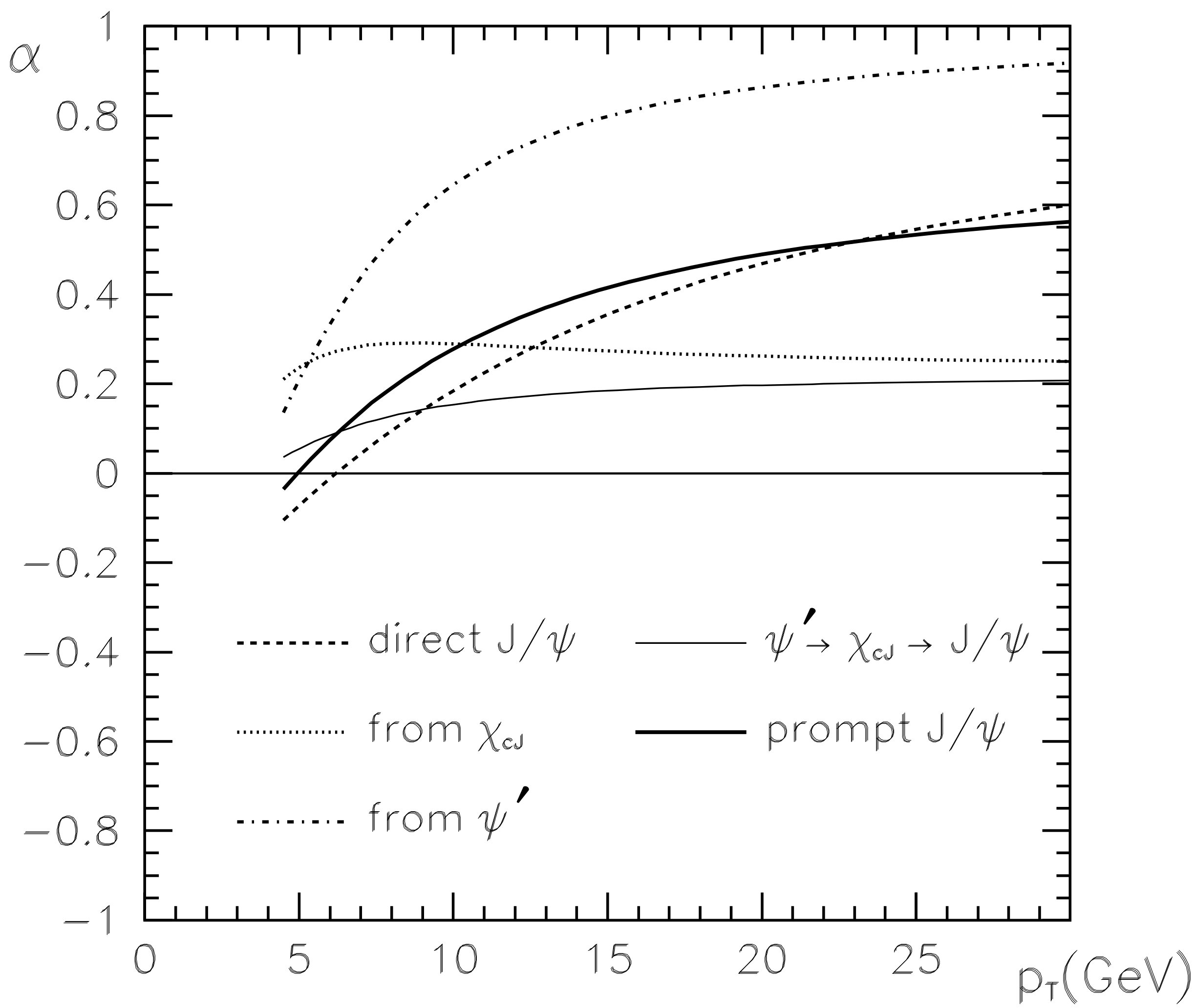

Figure 1: Polarization variable α as a function of p_T for direct J/ψ mesons, for those from single-cascade decays of χ_{cJ} and ψ' mesons, for those from double-cascade decays of ψ' mesons via χ_{cJ} intermediate states, and for prompt ones.